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Moshinsky Transformation and Slater Integral Methods as Evaluation Tools for Overlap Probability in Six-Quark Bag

Madhulika Mehrotra*
Department of Physics, Isabella Thoburn College, University of Lucknow, Lucknow, Uttar Pradesh, India

Abstract

A comparative study of Moshinsky transformation and Slater integral methods in successful calculation of Binding Energy of mirror hypernuclei pairs ($^{6}_{\Lambda}He \sim ^{6}_{\Lambda}Li$, $^{14}_{\Lambda}C \sim ^{14}_{\Lambda}N$), using six-quark probability of nucleon-nucleon ($P^{6q}_{NN}(r_0)$) and nucleon-Λ hyperon ($P^{6q}_{\Lambda N}(r_0)$). The contribution of direct and exchange terms to the six-quark probability show that the Pauli exchange terms in $P^{6q}_{NN}(r_0)$ is about 40% of the direct term, which leads to a sizable reduction in the six-quark probability. It is observed that the six-quark cluster formation probabilities obtained in Slater method are larger than the corresponding values obtained in Moshinsky method.

Keywords: *Moshinsky transformation, Slater integral, hypernuclei, binding energy difference of mirror hypernuclei pairs, six-quark probability*

***Author for Correspondence** E-mail: madhulikamehrotra@gmail.com

INTRODUCTION

During last few years, a great deal of attention has been made to the studies of the quark degree of freedom in nuclei and it has led to a better understanding of several nuclear phenomenon. Thus, invoking quark degrees of freedom in nuclei has a far-reaching consequence on several nuclear properties particularly nucleon-nucleon and nucleon-Λ hyperon interaction.

A number of deep inelastic scattering experiments of leptons and neutrino support the quark structure of hadrons. In the region where two hadrons overlap each other the internal structure of hadrons is expected to play more explicit role. The field of the strong interaction is the gluon field coupled to the color of quarks, not the pion field. The nuclear force is just the remainder of the strong force of the color neutral nucleon [1].

In the present work the six-quark bag contribution to the binding energy difference of $^{6}_{\Lambda}He - ^{6}_{\Lambda}Li$ and $^{14}_{\Lambda}C - ^{14}_{\Lambda}N$ has been estimated in the framework of hybrid quark model (HQM) [2] using Moshinsky transformation [3] and Slater integral methods [4]. The overlap probability of formation of a six-quark bag in the HQM, the two nucleons maintain their identity as long as the distance between them is greater than a certain cut off radius r_0. For distances smaller than r_0 the two baryons overlap and form a six-quark bag. Thus, HQM retains the conventional meson exchange picture at long distances and represents the effects of quantum chromodynamics (QCD) at short distances. Exact calculation of six-quark probability [5] within the constraints of QCD is difficult to make in a model independent way but it can be related to internal NN wave functions under different approximations. If the six-quark plus NN wave functions obey the same normalization conditions as an ordinary NN wave function, then the six-quark probability equals the probability defect of wave functions for $r < r_0$.

The probability to find six-quark for $r < r_0$ does not have to be the same as that of finding two nucleons for $r < r_0$. Using the Moshinsky transformation and Slater integral methods for the evaluation of matrix elements in the overlap probability of six-quark bag, a mathematical framework for nucleon-nucleon pair inside a nucleus or a hypernucleus from the shell model wave functions has been described in detail.

CALCULATION

The hybrid quark model can be generalized to the nucleon-nucleon pair inside a nucleus and the six-quark probability can be calculated from the shell model wave functions. We are interested with the state of the valence particle to form six-quark bag with the core particles only. The average probability $P_{NN}^{6q}(r_0)$ can be defined as sum of single particle terms as,

$$P_{NN}^{6q}(r_0) = \sum_{n_i l_i j_i t_i} (2j_i + 1) P_{n_i l_i j_i t_i}(r_0)$$

where,

$$P_{n_i l_i j_i t_i}(r_0) = \frac{1}{(2j_v + 1)(2j_i + 1)} \sum_{m_v m_i} \left\langle \phi_{\alpha_V}(1)\phi_{\alpha_i}(2) \middle| \theta(r_0 - r_{12}) \middle| \phi_{\alpha_V}(1)\phi_{\alpha_i}(2) - \phi_{\alpha_i}(1)\phi_{\alpha_V}(2) \right\rangle$$

$P_{n_i l_i j_i t_i}(r_0)$ is been interpreted as the probability for the valence particle to be within a distance of a specified core particle with quantum number $n_i l_i j_i t_i$. Where $\alpha_v(= n_v l_v j_v)$ and $\alpha_i(= n_i l_i j_i)$ represent the quantum states of the valence and the core nucleons respectively. Thus $P_{NN}^{6q}(r_0)$ can be expressed as a combination of a direct term $P_{n_i l_i j_i}^{d}(r_0)$ and an exchange term $P_{n_i l_i j_i}^{e}(r_0)$ as,

$$P_{NN}^{6q}(r_0) = \sum_{n_i l_i j_i} (2j_i + 1)\left[2P_{n_i l_i j_i}^{d}(r_0) - P_{n_i l_i j_i}^{e}(r_0)\right]$$

The valence nucleon can also overlap with the hyperon and from a six-quark bag with the hyperon. The corresponding overlap probability can be expressed as,

$$P_{\Lambda N}^{6q}(r_0) = P'_{n_0 l_0 j_0}(r_0)$$

With

$$P'_{n_0 l_0 j_0}(r_0) = \frac{1}{(2j_v + 1)(2j_0 + 1)} \sum_{m_n m_v} \left\langle \phi_{\alpha_0}(1)\phi_{\alpha_v}(2) \middle| \theta(r_0 - r_{12}) \middle| \phi_{\alpha_0}(1)\phi_{\alpha_v}(2) \right\rangle$$

There is no exchange term for ΛN overlapping. If instead of the average probability $P_{NN}^{6q}(r_0)$, we use the valence probabilities P_0, that it does not overlap with any of the nucleons or hyperons then, $P_{Q_1}^{6q}(r_0)$ for six-quark bag, $P_{Q_2}^{6q}(r_0)$ for nine-quark bag and $P_{\Lambda N}^{6q}(r_0)$; the completeness of the wave function demands,

$$P_0 + P_{Q_1}^{6q}(r_0) + P_{Q_2}^{6q}(r_0) + P_{\Lambda N}^{6q}(r_0) \cong 1$$

To calculate the overlap probability $P_{NN}^{6q}(r_0)$ and $P_{\Lambda N}^{6q}(r_0)$ for the evaluation of matrix elements using harmonic oscillator wave functions, following two methods are used: (1) Moshinsky transformation, and (2) Slater integral.

Moshinsky Transformation Method

In the Moshinsky method, the matrix elements in the overlap probability are transformed to the centre-of-mass and relative transformation coefficients and Moshinsky brackets, where the nucleons or hyperons in nuclei are described by harmonic oscillator wave functions. For calculation of Moshinsky Coefficient, consider the Hamiltonian of the two nucleons in a harmonic oscillator potential of frequency *w* as,

$$H(1,2)=\frac{1}{2}\left(p_1^2/m\right)+\frac{1}{2}mw^2r_1^2+\frac{1}{2}\left(p_2^2/m\right)+\frac{1}{2}mw^2r_2^2$$

The relative coordinate r and the centre-of-mass coordinate R may be defined as,

$$r=\frac{1}{\sqrt{2}}(r_1-r_2) \qquad R=\frac{1}{\sqrt{2}}(r_1+r_2)$$

The corresponding momentum is

$$p=\frac{1}{\sqrt{2}}(p_1-p_2)$$

$$P=\frac{1}{\sqrt{2}}(p_1+p_2)$$

The Hamiltonian can be written as,

$$H(1,2)=\frac{1}{2}\left(p^2/m\right)+\frac{1}{2}mw^2r^2+\frac{1}{2}\left(p^2/m\right)+\frac{1}{2}mw^2R^2$$

The angular momentum associated with the different coordinates will be designated by $l_1, l_2 l, L.$ from the conservation of angular momentum

$$l_1+l_2=\lambda=l+L$$

Both $\phi_{n_1l_1m_1}(r_1)\phi_{n_2l_2m_2}(r_2)$ and $\phi_{NML}(R)\phi_{nlm}(r)$ form a complete set of wave function of two particles moving in the harmonic oscillator potential. Thus, any one-product wave function $\phi_{n_1l_1m_1}(r_1)\phi_{n_2l_2m_2}(r_2)$ should be expressed in terms of a complete set of harmonic oscillator functions $\phi_{NML}(R)\phi_{nlm}(r)$. The wave function for a single harmonic oscillator will be given by

$$R_{nl}(r)Y_{lm}(\theta,\varphi)$$

Where Y_{lm} are spherical harmonics and $R_{nl}(r)$ the radial functions. As λ commutes with the Hamiltonian, the angular momentum coupled wave functions are defined as,

$$\left|n_1l_1,n_2l_2,\lambda\mu\right\rangle=\sum_{nlNL}\left\langle nl,NL;\lambda\middle|n_1l_1,n_2l_2,\lambda\right\rangle_{MB}$$

Where the quantity denoted as $\left\langle nl,NL,\lambda\middle|n_1l_1,n_2l_2,\lambda\right\rangle_{MB}$ is Moshinsky transformation brackets [6]. The LS coupled wave functions can be changed over to relative and centre-of-mass representation using Moshinsky transformation brackets. Thus,

$$\left|n_1l_1j_1,n_2l_2j_2;J\right\rangle=\sum_{\substack{\lambda S\\ nlNL}}A\begin{Bmatrix} l_1 & 1/2 & j_1\\ l_2 & 1/2 & j_2\\ \lambda & S & J\end{Bmatrix}nlNL;\lambda\left|n_1l_1n_2l_2;\lambda\right\rangle\left|nlNL;\lambda S;J\right\rangle$$

Then

$$\left\langle n_1l_1j_1,n_2l_2j_2;J\middle|V(r_1-r_2)\middle|n_1l_1j_1,n_2l_2j_2;J\right\rangle$$

$$=\sum_{\substack{\lambda S\\ nlNL}}A\begin{Bmatrix} l_1 & 1/2 & j_1\\ l_2 & 1/2 & j_2\\ \lambda & S & J\end{Bmatrix}\left[\left\langle nlNL;\lambda\middle|n_1l_1n_2l_2;\lambda\right\rangle\right]^2\left\langle nl\middle\|V(r_1-r_2)\middle\|nl\right\rangle$$

The final expression for the direct term is been simplified to,

$$P^d_{n_i l_i j_i}(r_0) = \sum_{\lambda SnlNLJM} A^2 \begin{bmatrix} l_i & 1/2 & j_i \\ l_i & 1/2 & j_v \\ \lambda & S & J \end{bmatrix} \langle nlNL;\lambda | n_i l_i n_v l_v;\lambda\rangle^2_{MB} \langle nl \| \theta(r_0 - r) \| nl\rangle$$

Where,

$$\langle nl \| \theta(r_0 - r) \| nl\rangle = \int_0^{r_0} R^2_{nl}(r)\,dr$$

$R_{nl}(r)$ is normalized radial functions and $\langle nlNL;\lambda | n_i l_i n_v l_v;\lambda\rangle_{MB}$ the transformation coefficients in the Moshinsky brackets. In calculating overlap probability of hyperons $P^{6q}_{\Lambda N}(r_0)$ the matrix elements of two baryons of different mass can be expanded in relative and centre-of- mass basis by

$$\nu_\Lambda = (m_\Lambda / m_N)\nu_N$$

Where ν_Λ and ν_N are the oscillator length parameter for hyperon and nucleon respectively.

Slater Integral Method

The direct and the exchange integral of potential $V(r_1 - r_2)$ can be solved by a technique developed (Slater, 1929) in atomic spectroscopy.

$$V(r_1 - r_2) = \sum_{\lambda=0}^{\infty} \vartheta_\lambda(r_1, r_2) P_\lambda(\cos\theta)$$

$P_\lambda(\cos\theta)$ Can be expanded in a finite set of spherical harmonics which are functions of $(\theta_1\varphi_1)$ and $(\theta_2\varphi_2)$ respectively and where,

$$\vartheta_\lambda(r_1 - r_2) = \frac{2\lambda+1}{2}\int_{-1}^{+1} V(r_1, r_2) P_\lambda(\cos\theta)(r_1, r_2)\, d(\cos\theta)$$

The integrals can be expressed as the products of radial and angular parts. Angular part can be integrated by using the standard techniques of angular momentum algebra. The radial integrals or Slater integrals F_λ in the direct matrix element are given by

$$F_1 = \int \left| \frac{Rn_1 l_1(r_1)}{r_1} \frac{Rn_2 l_2(r_2)}{r_2} \right|^2 \vartheta_\lambda(r_1, r_2) r_1^2 dr_1 r_2^2 dr$$

Similarly, the radial integrals in the exchange matrix element are

$$G_\lambda = \int \frac{Rn_1 l_1(r_1)}{r_1} \frac{Rn_1 l_1(r_1)}{r_1} \frac{Rn_2 l_2(r_2)}{r_2} \frac{Rn_2 l_2(r_2)}{r_2} \vartheta_\lambda(r_1, r_2) r_1^2 dr_1 r_2^2 dr_2$$

If the harmonic oscillator wave functions are used to describe the nucleons, the product of wave functions $\phi_{n_1 l_1 m_1}(r_1)\phi_{n_2 l_2 m_2}(r_2)$ can be transformed to a sum of products $\phi_{NLM}(R)\phi_{nlm}(r)$, with the following restrictions,

$(i) 2n_1 + l + 2n_2 + l_2 = 2n + l + 2N + L$, for energy conservation,

$(ii) l_1 + l_2 = \lambda = l + L$, for conservation of angular momentum. R and r refer to the centre-of-mass and relative coordinates of two nucleons and are defined as,

$$r = r_2 - r_1; 2R = r_2 + r_1$$
$$r_1^2 = R^2 + r^{2}\!/_4 - Rr\cos\alpha,$$
$$r_2^2 = R^2 + r^{2}\!/_4 + Rr\cos\alpha,$$
$$r_1.r_2 = r_1 r_2 \cos\theta = \frac{1}{4}\left(4R^2 - r^2\right)$$
$$r_1^2 + .r_2^2 = \frac{1}{2}\left(4R^2 + r^2\right)$$

For our calculation the function $\theta(r_0 - r_{12})$ is been expanded in complete set of Legendre polynomials of the angle θ between the vectors r_1 and r_2,

$$\theta(r_0 - r_{12}) = \sum_{\lambda=0}^{\infty} f_\lambda(r_1, r_2) P_\lambda(\cos\theta)$$

Where, $r = r_1 - r_2$

The unknown quantities $f_\lambda(r_1, r_2)$ are given in terms of $\theta(r_0 - r_{12})$ by the integral

$$f_\lambda(r_1, r_2) = \frac{2\lambda + 1}{2}\int_0^\infty \theta(r_0 - r_{12}) P_\lambda(\cos\theta) d(\cos\theta)$$

The direct and the exchange terms reduce to

$$P^d_{n_i l_i j_i}(r_0) = \tfrac{1}{2}\int_0^\infty dr_1 r_1^2 \int_0^\infty dr_2 r_2^2 \left|\phi_{n_i l_i j_i}(r_1)\phi_{n_v l_v j_v}(r_2)\right|^2 \int_{-1}^{1} d(\cos\theta)\theta(r_0 - r_{12})$$

And,

$$P^e_{n_i l_i j_i}(r_0) = \int_0^\infty dr_1 r_1^2 \int_0^\infty dr_2 r_2^2 e_{l_i j_i l_v j_v}(r_1, r_2)\phi^*_{n_i l_i j_i}(r_1)\phi^*_{n_i l_i j_i}(r_2)\phi^*_{n_v l_v j_v}(r_1)\phi^*_{n_v l_v j_v}(r_2)$$

Where,

$$e_{l_i j_i l_v j_v}(r_1, r_2) = (2l_v + 1)(2l_v + 1)\sum_\lambda (2l_v + 1)\left[\begin{pmatrix} l_i & l_v\,\lambda \\ 0 & 0\,0 \end{pmatrix}\begin{Bmatrix} \lambda & l_i\ l_v \\ 1/2 & j_v\ j_i \end{Bmatrix}\right]^2$$
$$\times \tfrac{1}{2}\int_{-1}^{1} d(\cos\theta)\theta(r_0 - r_{12}) P_\lambda(\cos\theta)$$

Where, $\phi_{n_i l_i j_i}(r_1)$ is the radial wave function for core nucleon and $\phi_{n_v l_v j_v}(r_2)$ valence nucleon function.

The formulae for the six-quark probability density for the mirror hypernuclei pair ${}^6_\Lambda He \sim {}^6_\Lambda Li$ and ${}^{14}_\Lambda C \sim {}^{14}_\Lambda N$ are given as follows:

${}^6_\Lambda He \sim {}^6_\Lambda Li$: In ${}^6_\Lambda He \sim {}^6_\Lambda Li$ the overlap probability is determined between $0s_{1/2}$ core nucleon and $0p_{1/2}$ valence nucleon. The direct and the exchange terms in the overlap probability are reduced.

$$P^d_{n_i l_i j_i}(r_0) = C_0\left[2(B1)T(1) + \left(\frac{A1}{2}\right)T(2)\right]$$

$$P^e_{n_i l_i j_i}(r_0) = \frac{C_0}{2}\left[2(B1)T(1) - \left(\frac{A1}{2}\right)T(2)\right]$$

Where,

$$C_0 = \left(\frac{16}{3}\right)\left(\frac{\nu_N^4}{\pi}\right)$$

$$A1=\int_0^{\infty} R^2 e^{-2\nu_N R^2}\,dR$$

$$B1=\int_0^{\infty} R^4 e^{-2\nu_N R^2}\,dR$$

$$T(1)=\int_0^{r_0} r^2 e^{-2\nu_N r^{1/2}}\,dr$$

$$T(2)=\int_0^{r_0} r^4 e^{-\nu_N r^{1/2}}\,dr$$

${}^{14}_{\Lambda}C\sim{}^{14}_{\Lambda}N$: In ${}^{14}_{\Lambda}C\sim{}^{14}_{\Lambda}N$ pair the six-quark formation probability for the valence nucleon can be expressed as the sum of the probability of the overlap of the $0s_{1/2}$ core nucleon and with $0p_{1/2}$ core nucleon.

$$P^{6q}_{NN}(r_0)={}^{I}P^{6q}_{NN}(r_0)+{}^{II}P^{6q}_{NN}(r_0)$$

${}^{I}P^{6q}_{NN}(r_0)$, is given by expression similar to direct and exchange terms in ${}^{6}_{\Lambda}He\sim{}^{6}_{\Lambda}Li$. ${}^{II}P^{6q}_{NN}(r_0)$, gets reduced to the following expressions,

$${}^{II}P^{d}_{n_il_ij_i}(r_0)=C_{00}\left[2(C1)T(1)+\left(\frac{A1}{4}\right)T(3)+\left(\frac{B1}{3}\right)T(2)\right]$$

$${}^{II}P^{e}_{n_il_ij_i}(r_0)=\frac{5C_{00}}{16}\left[16(C1)T(1)+(A1)T(3)-\left(\frac{40}{3}\right)(B1)T(2)\right]$$

Where

$$C_{00}=\left(\frac{32}{9}\right)\left(\frac{\nu_N^5}{\pi}\right)$$

$$C1=\int_0^{\infty} R^6 e^{-2\nu_N R^2}\,dR$$

$$T(3)=\int_0^{\infty} r^6 e^{-\nu_N r^{1/2}}\,dr$$

RESULTS AND DISCUSSION

In the present work we have estimated the six-quark probability on the binding energy difference of mirror pair of p-shell hypernuclei (${}^{6}_{\Lambda}He$ □ ${}^{6}_{\Lambda}Li$, ${}^{14}_{\Lambda}C$ □ ${}^{14}_{\Lambda}N$). We have discussed the various results obtained in the $\Lambda-$binding energy difference of mirror hypernuclei pair ${}^{6}_{\Lambda}He-{}^{6}_{\Lambda}Li$ and ${}^{14}_{\Lambda}C-{}^{14}_{\Lambda}N$. The six-quark probabilities needed in the evaluation have been calculated using harmonic oscillator wave functions. The results for the six-quark probabilities and their contribution to the binding energy difference for different choices of the various parameters have been summarized in Tables 1–. The results show that the six-quark bag formation probabilities for both NN and ΛN overlaps are strongly dependent on the choice of the parameters. NN overlap probabilities range between 3% to 10 % and ΛN overlap probabilities lies between 0.4% and 1%, for r_0 = 1 fm. The variation of six-quark probabilities with oscillator length parameters for A=6 and 14 hypernuclei has been shown in Figures 1and 2.

The results of our calculations show that the six-quark bag formation effect contributes significantly to the binding energy difference of the mirror pair of nuclei. The contribution ranges from 14 keV to 157 keV for ${}^{6}_{\Lambda}He\sim{}^{6}_{\Lambda}Li$ and 38 keV to 203 keV for ${}^{14}_{\Lambda}C\sim{}^{14}_{\Lambda}N$. The calculations show that the overlap probability of the hyperon with the valence nucleon makes a smaller contribution to the binding energy difference. It is also observed that six-quark cluster formation effect increases the binding of Λ- hyperon in the neutron rich nuclei compared to that of its proton rich nuclei. The Λ- particle is more bound in ${}^{6}_{\Lambda}He,{}^{14}_{\Lambda}C$ compared to that in ${}^{6}_{\Lambda}Li,{}^{14}_{\Lambda}N$ respectively.

In the calculation of six-quark probabilities by Slater method the values of ν_N and ν_Λ can be fixed independently of each other. However, in Moshinsky method, to facilitate Moshinsky transformation to relative basis in the matrix elements in given equation, we have to choose $\nu_N=(m_\Lambda/m_N)\nu_N$. Thus fixing ν_N automatically fixes ν_Λ and vice versa. This prescription has been used earlier by Bando et al (1985) [7] and Mehrotra (1991) [8] in the study of hypernuclei. In the calculations of Mujib I and Mujib II [9], the oscillator length parameters ν_Λ and ν_N are obtained by fixing the value of one of the oscillator length parameters ν_Λ or ν_N and calculating the other from the above prescription [10]. For ${}^{6}_{\Lambda}He\sim{}^{6}_{\Lambda}Li$ pair the results of our calculation for six-quark probabilities using Moshinsky method are shown in Table 1 and for Slater method in Table 2 for the parameter of Gal I

[11]. In these Tables 1 and 2, $P_{Q_1}^{6q}(r_0)$ and $P_{Q_2}^{6q}(r_0)$ give the values of the exclusive probabilities for the formation of six-quark bags $P_{Q_1}^{6q}(r_0)$ and nine-quark bags $P_{Q_2}^{6q}(r_0)$ of the valence nucleon with one core nucleon and two core nucleons respectively.

Table 1: *In $^{6}_{\Lambda}He \sim ^{6}_{\Lambda}Li$ the Average Probability of the Valence Nucleon, to Form a Six-quark Bag with the Core Nucleons $P_{NN}^{6q}(r_0)$ and Hyperon $P_{\Lambda N}^{6q}(r_0)$ for Different Values of the Cut Off Radius r_0. $P_{Q_1}^{6q}(r_0)$ and $P_{Q_2}^{6q}(r_0)$ are the Six-quark and Nine-quark Bag Formation Probabilities with One and Two Core Nucleons Respectively. The Values Shown Have Been Calculated with the Parameters of Gal I using Moshinsky Method (In Gal I, $\nu_N = 0.41 fm^{-2}$; $\nu_\Lambda = 0.49 fm^{-2}$)*

r_0 (fm)	$P_{NN}^{6q}(r_0)$	$P_{Q_1}^{6q}(r_0)$	$P_{Q_2}^{6q}(r_0)$	$P_{\Lambda N}^{6q}(r_0)$
0.85	0.03105	0.03033	0.000119	0.00439
0.87	0.03668	0.03568	0.000165	0.00520
0.89	0.03673	0.03573	0.000166	0.00521
0.91	0.04300	0.04163	0.000226	0.00612
0.93	0.04306	0.04168	0.000227	0.00613
0.95	0.04311	0.04173	0.000228	0.00614
0.97	0.04994	0.04809	0.000304	0.00714
0.99	0.04999	0.04814	0.000305	0.00715
1.0	0.05004	0.04819	0.000306	0.00716

Table 3 (for Moshinsky method) and Table 4 (for Slater method), show the contribution of direct and exchange terms to the six-quark probability. It is worth noting that the Pauli exchange terms in $P_{NN}^{6q}(r_0)$ is about 40% of the direct term, which leads to a sizable reduction in the six-quark probability.

The six-quark probability $P_{NN}^{6q}(r_0)$ and $P_{\Lambda N}^{6q}(r_0)$ depend on the cut off radius r_0. Thus, it is necessary to use some constrain for the value of r_0, hence the results for the six-quark probabilities for other sets of oscillator length parameters for Moshinsky and Slater method are shown in Tables 5 and 6 respectively for $r_0 = 1 fm$.

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Table 2: *In $^{6}_{\Lambda}He \sim ^{6}_{\Lambda}Li$ the Six-quark Probabilities $P_{NN}^{6q}(r_0)$, $P_{Q_1}^{6q}(r_0)$, $P_{\Lambda N}^{6q}(r_0)$ and Nine-quark Bag Probabilities $P_{Q_2}^{6q}(r_0)$ for Different Values of the Cut Off Radius r_0. The Values Shown Have been Calculated with the Parameters of Gal I using **Slater Method** (In Gal I, $\nu_N = 0.41 fm^{-2}$; $\nu_\Lambda = 0.49 fm^{-2}$).*

r_0 (fm)	$P_{NN}^{6q}(r_0)$	$P_{Q_1}^{6q}(r_0)$	$P_{Q_2}^{6q}(r_0)$	$P_{\Lambda N}^{6q}(r_0)$
0.85	0.05758	0.05513	0.000403	0.01915
0.87	0.06854	0.06508	0.000567	0.02271
0.89	0.06866	0.06518	0.000569	0.02274
0.91	0.08108	0.07625	0.000789	0.02674
0.93	0.08120	0.07636	0.000791	0.02678
0.95	0.08133	0.07647	0.000794	0.02682
0.97	0.09505	0.08844	0.001076	0.03121
0.99	0.09518	0.08854	0.001079	0.03124
1.0	0.09531	0.08866	0.001082	0.03128

Table 3: *In $^{6}_{\Lambda}He \sim ^{6}_{\Lambda}Li$ the Contribution of Direct and Exchange Terms to the Average Six-Quark Bag Probabilities $P_{NN}^{6q}(r_0)$ For the Valence Nucleon to Form a Six-quark Bag with the Core Nucleon, as a Function of Cut Off Radius r_0. The Values Shown Have been Calculated with Gal I Parameters using Moshinsky Method.*

r_0 (fm)	**Direct term**	**Exchange term**
0.85	0.07904	0.03387
0.87	0.09336	0.04001
0.89	0.09349	0.04007
0.91	0.10947	0.04691
0.93	0.10960	0.04697
0.95	0.10973	0.04703
0.97	0.12712	0.05448
0.99	0.12725	0.05453
1.0	0.12738	0.05459

The six-quark probability $P_{NN}^{6q}(r_0)$ is strongly dependent on the choice of the parameters and ranges between 3% to 10% for r_0 = 1.0 fm. $P_{\Lambda N}^{6q}(r_0)$ is much smaller and lies between 0.4% to 1%. In all the calculations, the six-quark cluster formation probabilities obtained in Moshinsky method are smaller than the corresponding values obtained in Slater method. This is in accordance with the calculation of Kang and Oshagan [12]. These authors have shown that the value of six-quark probability calculated by using Moshinsky method is smaller than those calculated by Slater method. The results of our calculation show that the quark contribution to the binding energy is sizable and varies between 14 keV to 157 keV. The split is various terms contributing to the quark correction to the binding energy difference is shown in Table 7 for the parameters of Gal I [10].

Table 4: *In* ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ *the Contribution of Direct and Exchange Terms to the Average Six-Quark Bag Probabilities* $P^{6q}_{NN}(r_0)$ *for the Valence Nucleon to form a Six-quark Bag with the Core Nucleon, as a Function of Cut Off Radius* r_0*. The Values shown have been Calculated with Gal I Parameters using Slater Method.*

r_0 (fm)	**Direct term**	**Exchange term**
0.85	0.01855	0.00832
0.87	0.02201	0.00975
0.89	0.02204	0.00976
0.91	0.02593	0.01132
0.93	0.02597	0.01133
0.95	0.02600	0.01134
0.97	0.03028	0.01302
0.99	0.03031	0.01303
1.0	0.03035	0.01304

Table 5: *In* ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ *Average Six-quark Probability for the Valence Nucleon to Form a Six-Quark Bag with the Core Nucleons* $P^{6q}_{NN}(r_0)$ *and Hyperon* $P^{6q}_{\Lambda N}(r_0)$ *for the Cut Off Radius* $r_0 = 1 fm$. $P^{6q}_{Q_1}(r_0)$ *and* $P^{6q}_{Q_2}(r_0)$ *are the Six-quark and Nine-quark Bag Formation Probabilities with One and Two Core Nucleons Respectively. The Values Shown Have been Calculated in Moshinsky Method for Different Sets of Oscillator Length Parameters* (ν_N, ν_Λ)*.*

Reference	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$
Mujib I[a]	0.02793	0.02735	0.00010	0.00393
Mujib II[b]	0.07969	0.07502	0.00076	0.01156
Wang[c]	0.03572	0.03477	0.00016	0.00504

a) $\nu_N = 0.272\, fm^{-2}$; $\nu_\Lambda = 0.1477\, fm^{-2}$.

b) $\nu_N = 0.5746\, fm^{-2}$; $\nu_\Lambda = 0.312\, fm^{-2}$.

c) $\nu_N = 0.323\, fm^{-2}$; $\nu_\Lambda = 0.384\, fm^{-2}$.

Table 6: *For* ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ *Six-quark Bag Probabilities* $P^{6q}_{NN}(r_0)$, $P^{6q}_{Q_1}(r_0)$, $P^{6q}_{\Lambda N}(r_0)$ *and Nine-Quark Bag Probabilities* $P^{6q}_{Q_2}(r_0)$ *Obtained by Slater Method for the Cut Off Radius* $r_0 = 1 fm$. *The Values Shown Have Been Calculated for Different Sets of Oscillator Length Parameters* ν_N, ν_Λ *Used by Different Authors.*

Reference	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$
Mujib I[a]	0.09531	0.08866	0.00108	0.02845
Mujib II[b]	0.05153	0.04956	0.00032	0.01709
Wang[c]	0.06668	0.06340	0.00054	0.02209

a) $V_N = 0.41 fm^{-2}$; $V_\Lambda = 0.33 fm^{-2}$.

b) $V_N = 0.272 fm^{-2}$; $V_\Lambda = 0.312 fm^{-2}$.

c) $V_N = 0.323 fm^{-2}$; $V_\Lambda = 0.384 fm^{-2}$.

Table 7: *For* ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$*, Split of Various Terms in the Quark Contribution to the* $\Lambda -$ *Binding Energy Difference* (ΔB_{6q}) *of* $r_0 = 1 fm$*. The Values Shown Have Been Calculated using Gal I Parameters and NRQM I Model for Mass Difference of Six-Quark Bag of Two Neutrons and Two Protons.*

Terms Contributing to ΔB_{6q}	**NRQM I**	
	Moshinsky Method (MeV)	*Slater Method (MeV)*
$\frac{1}{2} P^{6q}_{NN}(r_0)(2m_n - 2m_p)$	0.0651	0.1239
$\frac{1}{2} P^{6q}_{NN}(r_0)(m_{pp} - m_{nn})$	+0.0465	+0.0886
$P^{6q}_{\Lambda N}(r_0)(m_n - m_p)$	+0.0093	+0.0407
$P^{6q}_{\Lambda N}(r_0)(m_{p\Lambda} - m_{n\Lambda})$	-0.0100	-0.0438

The contribution in the binding energy difference due to six-quark and nine-quark bags is 0.0522 MeV and 0.0004 MeV respectively in the Moshinsky method. The corresponding values are 0.0961 MeV and 0.0015 MeV in Slater method. Thus, the dominant contribution to the binding energy difference comes from the exclusive six-quark probability.

We have made similar calculation for ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ hypernuclei pair. The results for the six-quark bag formation probabilities and various other terms are summarized in Table 8–11 for different cases for both Moshinsky and Slater methods.

Table 8: *In* ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ *Average Probabilities for the Valence Nucleon to Form Six-quark Bag with the Core Nucleons* $P^{6q}_{NN}(r_0)$ *and Hyperon* $P^{6q}_{\Lambda N}(r_0)$ *for Different Values of the Cut Off Radius r₀.* $P^{6q}_{Q_1}(r_0)$ *and* $P^{6q}_{Q_2}(r_0)$ *are the Six-quark and Nine-quark Bag Formation Probabilities with One and Two Core Nucleons Respectively. The Values Shown Have been Calculated with the Parameters of Gal I using Moshinsky Method. (In Gal I,* $\nu_N = 0.41 fm^{-2}; \nu_\Lambda = 0.49 fm^{-2}$*).*

r_0 (fm)	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$
0.85	0.06744	0.06339	0.000179	0.00879
0.87	0.07989	0.07423	0.000248	0.01041
0.89	0.08000	0.07433	0.000249	0.01043
0.91	0.09395	0.08617	0.000340	0.01225
0.93	0.09407	0.08627	0.000341	0.01226
0.95	0.09420	0.08638	0.000342	0.01228
0.97	0.10946	0.09896	0.000455	0.01428
0.99	0.10958	0.09906	0.000456	0.01429
1.0	0.10970	0.09916	0.000457	0.01431

Table 9: *In* ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ *Six-quark Probabilities* $P^{6q}_{NN}(r_0)$, $P^{6q}_{Q_1}(r_0)$, $P^{6q}_{\Lambda N}(r_0)$ *and Nine-quark Probabilities* $P^{6q}_{Q_2}(r_0)$ *are Obtained by Slater Method for Different Cut Off Radius r₀ with the Parameters of Gal.*

r_0 (fm)	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$
0.85	0.07678	0.07154	0.000230	0.01915
0.87	0.09385	0.08608	0.000339	0.02271
0.89	0.09410	0.08630	0.000341	0.02274
0.91	0.11430	0.10288	0.000495	0.02674
0.93	0.11458	0.10311	0.000497	0.02678
0.95	0.11488	0.10334	0.000499	0.02682
0.97	0.13799	0.12150	0.000707	0.03121
0.99	0.13830	0.12174	0.000710	0.03124
1.0	0.13862	0.12199	0.000713	0.03128

Table 10: *In* ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ *Average Probabilities for the Valence Nucleon to form Six-quark Bag with the Core Nucleons* $P^{6q}_{NN}(r_0)$ *and* $P^{6q}_{\Lambda N}(r_0)$ *Hyperon for the Cut Off Radius* $r_0 = 1 fm$. *The Values Shown Have been Obtained Using Different Sets of Oscillator Length Parameter* (ν_N, ν_Λ) *in Moshinsky Method.*

Reference	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$
Mujib I[a]	0.09355	0.08583	0.000337	0.01217
Mujib II[b]	0.09873	0.09016	0.000374	0.01285
Wang[c]	0.10023	0.09139	0.000385	0.01303

a) $\nu_N = 0.367 fm^{-2}; \nu_\Lambda = 0.1993 fm^{-2}$.

b) $\nu_N = 0.381 fm^{-2}; \nu_\Lambda = 0.207 fm^{-2}$.

c) $\nu_N = 0.385 fm^{-2}; \nu_\Lambda = 0.209 fm^{-2}$.

Table 11: *For* ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ *Six-quark Probabilities* $P^{6q}_{NN}(r_0)$, $P^{6q}_{\Lambda N}(r_0)$, $P^{6q}_{Q_1}(r_0)$, *and Nine-Quark Probabilities* $P^{6q}_{Q_2}(r_0)$ *are Obtained by Slater Method for Different Sets of Oscillator Length Parameters* (ν_N, ν_Λ) *used by Different Authors at Cut Off Radius* $r_0 = 1 fm$.

Reference	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$
Mujib I[a]	0.13862	0.12199	0.000713	0.02845
Mujib II[b]	0.11388	0.10254	0.000491	0.02040
Wang[c]	0.12396	0.11058	0.000577	0.02853

a) $\nu_N = 0.41 fm^{-2}; \nu_\Lambda = 0.33 fm^{-2}$.

b) $\nu_N = 0.367 fm^{-2}; \nu_\Lambda = 0.207 fm^{-2}$.

c) $\nu_N = 0.385 fm^{-2}; \nu_\Lambda = 0.4574 fm^{-2}$.

$P^{6q}_{NN}(r_0)$ and $P^{6q}_{\Lambda N}(r_0)$ are dependent on the choice of oscillator length parameters and increase with an increase of the values of ν_N and ν_Λ. Figures 1 and 2 show the variation of $P^{6q}_{NN}(r_0)$ with ν_N, for A=6 and A=14 mirror hypernuclei pair respectively. The variation is shown for the parameters of Gal I for both Moshinsky and Slater methods. In the case of ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ pair, overlap probability of the valence nucleon with the $0s_{1/2}$ core nucleon and $0p_{3/2}$ core nucleons. As expected for $0p_{1/2}$ valence nucleon, overlap probability with the s-core nucleons is larger, compared to that with the p-core nucleons. The results of our calculation show that the overlap probability of the valence nucleon with the hyperon also make a small contribution to the binding energy difference and the six-quark bag formation effect contributes significantly to the binding energy difference of the mirror pair of nuclei.

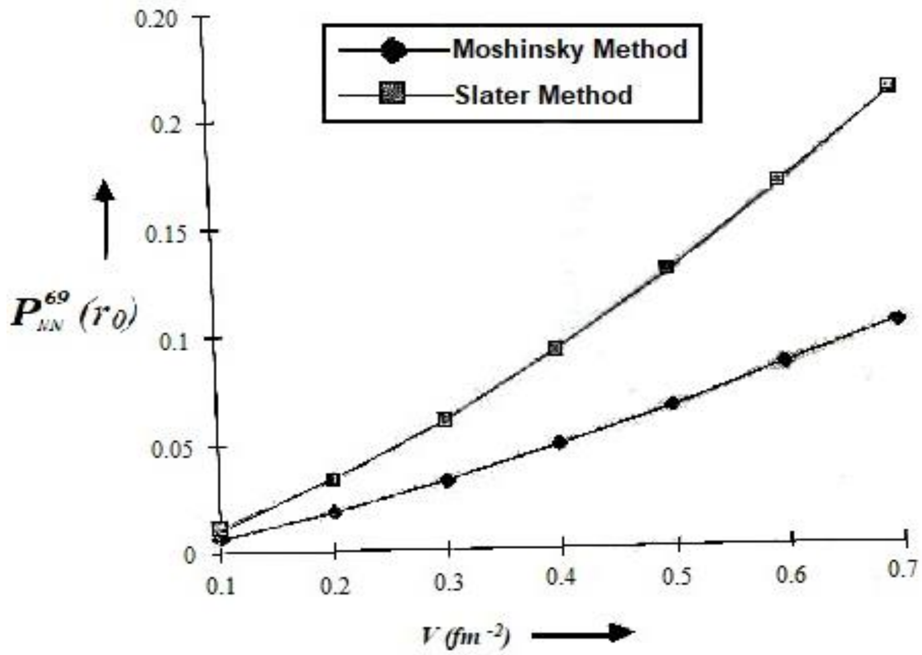

Fig. 1: *Graph Showing Variation of Six-quark Probability* $P_{NN}^{6q}(r_0)$ *with Oscillator Length Parameter* V_N *for A = 6 Hypernuclei with Cut Off Radius* r_0 *= 1 fm.*

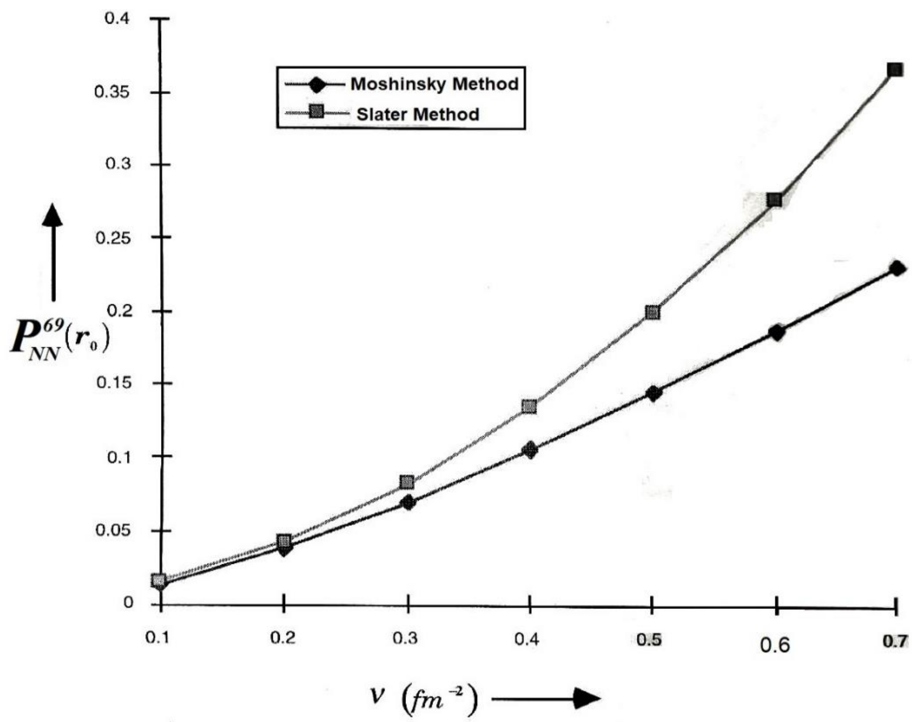

Fig. 2: *Graph Showing Variation of Six-quark Probability* $P_{NN}^{6q}(r_0)$ *with Oscillator Length Parameter* V_N *for A = 14 Hypernuclei with Cut Off Radius* r_0 *= 1 fm.*

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CONCLUSION

(i) The six-quark probabilities evaluated separately for Moshinsky and Slater methods for r_0=1.0 fm, in Figures 1 and 2 for A= 6, 14 hypernuclei, show the variation of oscillator length parameter (ν) from 0.1 fm^{-2} to 0.7 fm^{-2}.

(ii) Our observations show that the values of six-quark probability calculated by using Moshinsky method are smaller than those calculated by Slater method.

(iii) The contribution of direct and exchange terms to the six-quark probability show that the Pauli exchange terms in $P_{NN}^{6q}(r_0)$ is about 40% of the direct term, which leads to a sizable reduction in the six-quark probability, as calculated by Moshinsky and Slater methods.

(iv) The six-quark probability $P_{NN}^{6q}(r_0)$ is strongly dependent on the choice of the oscillator length parameters, which ranges from 3% to 10% for both methods.

(v) The six-quark probability $P_{\Lambda N}^{6q}(r_0)$ lies from 0.4% to 1% for r_0 = 1.0 fm which is much smaller than $P_{NN}^{6q}(r_0)$ for both methods.

(vi) Our calculation shows that the binding energy difference of the mirror pair of nuclei with six-quark bag formation effect using Moshinsky and Slater methods, contributes significantly for ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ and ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$.

(vii) The contributions in the binding energy difference, due to six-quark and nine-quark bags are 0.0522 MeV and 0.0004 MeV respectively, in the Moshinsky method and the corresponding values are 0.0961 MeV and 0.0015 MeV in Slater method. Thus, the dominant contribution to the binding energy difference comes from the exclusive six-quark probability.

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